

Faster Than Light ?

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Abstract

In this paper we present a pedestrian review of the theoretical fact that all relativistic wave equations possess solutions of arbitrary velocities $0 \leq v < \infty$. We discuss some experimental evidences of $v \geq c$ transmission of electromagnetic field configurations and the importance of these facts with regard to the principle of relativity.

Maxwell's equations are a set of first order partial differential equations describing the behavior of the electric and magnetic fields generated by distributions of charges and currents. Such equations also possess solutions for the case where there are no charges nor currents. The simplest solutions of this kind, discovered by Maxwell himself, describe electromagnetic field configurations which we will call *light-solutions* (*LS*). Light-solutions propagate in empty space with a particular velocity c ($c \approx 300,000$ km/s in MKS units). The simplest, easiest to find *LS*s are the plane wave solutions (*PW*), which have the following important characteristics:

- (i) *PW*s are *transverse* waves: the electric (\vec{E}) and magnetic (\vec{B}) fields oscillate in time with a certain frequency and are perpendicular to the direction of propagation and to each other.
- (ii) The field invariants of *PW*s are null. In adequate units, where $c = 1$, these field invariants are given by $I_1 = \vec{E} \cdot \vec{B}$ and $I_2 = \vec{E}^2 - \vec{B}^2$.

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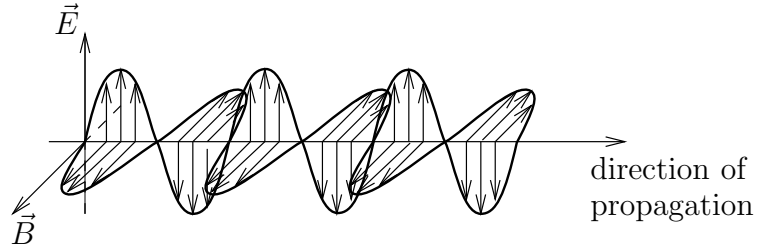


Figure 1: Representation of the oscillating electric (\vec{E}) and magnetic (\vec{B}) fields of a plane wave.

Figure 1 is a pictorial representation of the electric and magnetic fields of a plane wave.

One cannot elude a very natural question about *LSs*: What is the reference frame with respect to which *LSs* propagate with velocity c ? For Maxwell and his contemporaries the answer was that *LSs* have velocity c in a frame “materialized” by a special medium called *aether*, which is their “carrier”, i.e. *LSs* are vibrations of this aether. The aether concept was, at their time, an important part of electromagnetic theory, as indissociable of it as the concepts of charge and current. The advent of special relativity, and above all of Einstein’s interpretation for it, has taken the aether concept out of mainstream physics and into the books on history of science.¹

The Earth, our cosmic home, is endowed with several forms of motion more or less easily detectable. One such motion is its translation *around* the Sun. It thus cannot be permanently at rest with respect to the aether and many experiments (of which the most famous is that of Michelson and Morley) have been carried in order to detect the effect of the Earth’s motion on the measured velocity of light, without success. These negative results ultimately led to the formulation of special relativity by Lorentz, Poincaré and Einstein.

Before we continue it is important to examine carefully the concept of velocity itself. Note that in order to determine experimentally the mean velocity of a particle that traverses a distance L in such a way that our experiment is a realization of the mathematical definition of this concept, we must measure the distance L with standard rules and we shall also need at least two standard clocks. One of them will be at the beginning of the

¹Contrary to what is written in most textbooks, Einstein never really abandoned the aether concept, and has even written several articles in which he claims that this concept is an essential one. See in this respect the article by L. Kostro [1].

path L and the other at the end. The clocks must be synchronized, that is, we must be sure that when the first clock displays a time, say, $t = 0$, the other will register the same time.

Probably the most noticeable contribution of Einstein to special relativity was the realization that such a synchronization process depended on a definition. Einstein then adopted a definition (called synchronization ‘à l’Einstein’) that took into account the empirical fact that the time for a light signal to go from a point A to a point B in space, with both points fixed in a given inertial reference frame,² is independent of the velocity of the source that emits the signal. He proposed that the clocks at A and B (in any inertial reference frame) should be synchronized in the following way: The observer in A sends a light signal to B , where the signal is instantaneously reflected back to A . The observer in A determines the total traversal time τ for the path $A \rightarrow B \rightarrow A$, and asks the observer in B to adjust his clock in such a way that it would show a time $\tau/2$, i.e. one-half the total time for the path $A \rightarrow B \rightarrow A$, at the moment when the light signal arrived there. It can be shown that this definition implies that the measured velocity of light will be always c , in any reference frame, once the standard rules and clocks have a behavior different from that presupposed by classical theory. And in fact such a distinct behavior is exactly that found in nature, with a very good approximation. In particular, it has been found empirically by the American physicists J. C. Hafelle and T. E. Keating, in 1968 [3], that when two atomic clocks are synchronized at a given point of space in an inertial reference frame, if one of them makes a trip and comes back to the initial point after some time, then it will register a time interval smaller than the time measured by the clock that remained at rest, and the difference will be exactly what is needed in order that the measurement of the velocity of any light-solution will be c in every inertial reference frame.³ If in two inertial reference frames the spacetime coordinates of events are determined by standard rules and standard clocks synchronized ‘à l’Einstein’, then the coordinates of an event as determined in both reference frames will be related by the famous Lorentz transformations (fig. 2⁴).

All these ideas were incorporated by Poincaré, in 1904, and Einstein, in 1905, in what is now known as the theory of special relativity. Poincaré (and also Einstein) assumed the validity of a universal principle, called principle of relativity, which establishes that the development of all natural phenom-

²For a rigorous definition of inertial frame, see [2].

³In fact, Hafelle and Keating did the experiment on Earth, which is a non inertial frame. A full account of their results requires general relativity to be well understood.

⁴Adapted from [4].

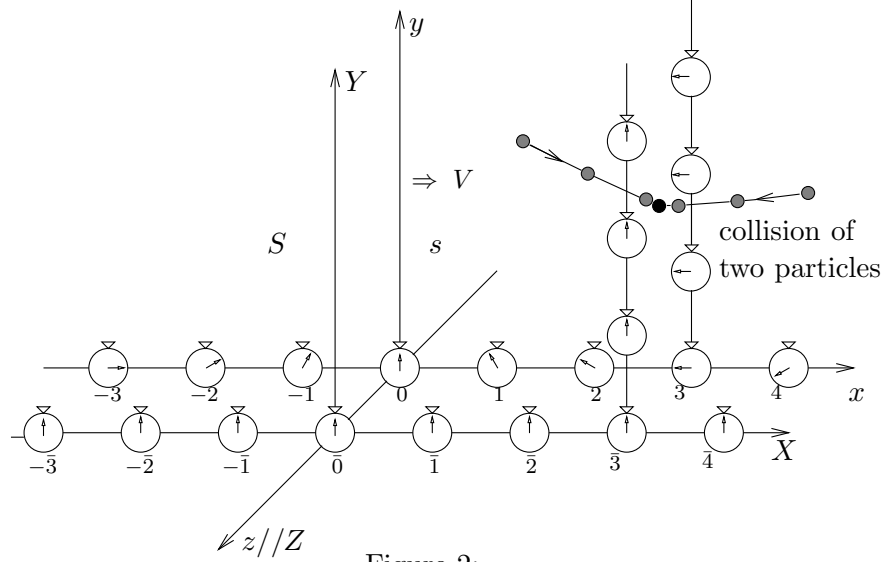


Figure 2:

How is it possible that the velocity of light is always c in every inertial reference frame? Suppose that S and s are two inertial laboratories that move with velocity V with respect to each other. Both laboratories are equipped with standard clocks synchronized à l'Einstein, and the origins of their coordinate systems coincide for $T = t = 0$. In S , the equation of motion for a light signal emitted at $T = t = 0$ is written $c^2 T^2 - X^2 - Y^2 - Z^2 = 0$. In s , the equation for the same signal is $c^2 t^2 - x^2 - y^2 - z^2 = 0$. These equations (together with a few other reasonable hypotheses) imply that the relation between t and T , on one hand, and between x and X , on the other, cannot be those used in the Newtonian theory of spacetime. Indeed, it can be shown that the transformations relating the coordinates of any event e (e.g. the collision of two particles) in systems S and s , and such that both equations for the light signals are true, are

$$t = \frac{T - VX/c^2}{\sqrt{1 - V^2/c^2}}, \quad x = \frac{X - VT}{\sqrt{1 - V^2/c^2}}, \quad y = Y, \quad z = Z;$$

$$T = \frac{t + Vx/c^2}{\sqrt{1 - V^2/c^2}}, \quad X = \frac{x + Vt}{\sqrt{1 - V^2/c^2}}, \quad Y = y, \quad Z = z.$$

Notice that the coordinates of event e are $e = (T, X, Y, Z)$ in S and $e = (t, x, y, z)$ in s . These are the famous Lorentz transformations, which allow us to calculate, from the measurements made by a certain observer, the results that would be obtained by another observer whose state of motion relative to the first one is known, if he observed the same phenomenon. For $c \rightarrow \infty$ (or for $V/c \ll 1$) these transformations reduce to the Galilean transformations ($x = X - VT$, $t = T$, $y = Y$, $z = Z$) used in Newtonian theory of spacetime and in our quotidian calculations, which involve systems endowed with velocities much smaller than the velocity of light.

ena does not depend on the state of uniform motion of the inertial reference frame where they take place.⁵ It can be shown that, from the mathematical point of view, this implies that all natural phenomena shall be described by equations which possess the Lorentz group as their symmetry group. Moreover, one can show that the validity of the principle of relativity implies that no internal process of synchronization of clocks in a given inertial frame (i.e. synchronization without “looking outside the laboratory”) will differ from the synchronization à l’Einstein [6].

About his attempt to formulate the principle of relativity, Einstein says in his *Autobiographical Notes*: “After ten years of reflection such a principle resulted from a paradox upon which I had already hit at the age of sixteen: If I pursue a beam of light with the velocity c (velocity of light in a vacuum), I should observe such a beam of light as a spatially oscillatory electromagnetic field at rest. However, there seems to be no such thing, whether on the basis of experience or according to Maxwell’s equations.”

Well, the fact is that Einstein was mistaken. Maxwell’s equations are a source of big surprises. Indeed, just ten years after the publication of Einstein’s fundamental article, the American mathematician H. Bateman, showed in his book *Electrical and Optical Motion* [7] that the scalar wave equation has solutions that describe a non spreading packet that travels with speed less than that of any LS ! Also, the late professor A. O. Barut, of Boulder University, published in 1992 an extraordinary article [8] where he showed that Maxwell’s equations without sources also possess wave packet solutions that travel with velocities less than that of LS s. These solutions present a little dispersion, but it can be shown that in many cases the time for the spreading of the packet is comparable to the presumed age of the universe. Such packets might eventually be used to represent elementary particles, which would turn out to be nothing but special electromagnetic configurations. This idea was developed in [9].

But do Maxwell equations predict the existence of any electromagnetic field configuration that propagates with a velocity greater than c ? The surprising answer to this question is *yes*. It has been recently proved [10, 9, 11] that all relativistic wave equations—the scalar wave equation, Maxwell’s equations, and the Klein-Gordon, Dirac and Weyl equations—have solutions that propagate with arbitrary velocities $0 \leq v < \infty$. It has been verified that Maxwell’s equations have, besides LS s, solutions corresponding to electromagnetic field configurations that propagate with superluminal

⁵The precedence of Poincaré over Einstein is well documented. The interested reader may look at [5] for details.

velocities in vacuum.⁶ One particular superluminal solution is the so called electromagnetic X wave, which does not distort as it propagates. Figure 3 shows the real part of the X wave solution for the homogeneous wave equation on the plane $y = 0$, propagating in the z direction. A complete solution of Maxwell equations may be obtained from it by the Hertz potential method [9]. In general, electromagnetic configurations that move (in vacuum) with velocities $v \neq c$ have a longitudinal component (electric and/or magnetic) and possess at least one non null field invariant. The exact electromagnetic X wave has infinite energy (as is the case for plane wave solutions) and thus cannot be produced in practice. Nevertheless, computer simulations of *finite aperture approximations* for the X wave, which have finite energies, have shown that these approximations also propagate with velocity greater than that of LS s. These approximate solutions distort a little as they propagate. Fig. 4 shows the results of such simulations for two different approximations for an X wave.

Nobody has so far produced an approximate electromagnetic X wave. However, there are serious reasons to believe that this is possible. One of them is the production of (approximate) acoustic X waves, described in the article by Rodrigues and Lu [9]. Acoustic waves satisfy a scalar wave equation where the parameter c is replaced by c_s , the velocity of sound. It has indeed been verified that acoustic X waves can travel with speeds greater than c_s . Also, another kind of non spreading acoustic waves called *Bessel pulses* were produced, which traveled with velocity less than c_s , as predicted by the theory (figs. 5 and 6) .

One may also find theoretical predictions of superluminal propagation of electromagnetic configurations in several situations which are mathematically modelled through boundary value problems. As examples we cite:

- (i) Maxwell's equations, when we take into account the quantum theory of fields. If one solves these equations for the electromagnetic field inside the region limited by two perfectly reflecting mirrors with conducting surfaces, one verifies that the velocity of the electromagnetic field is greater than that of LS s (in vacuum) in the direction perpendicular to the surface of the mirrors. This solution has been found by G. Barton and K. Scharnhorst [13].
- (ii) One can show that under appropriate boundary conditions it is possible to generate wave packets that propagate with superluminal velocities outside a conducting cylinder. This result has been discovered by

⁶See also [12].

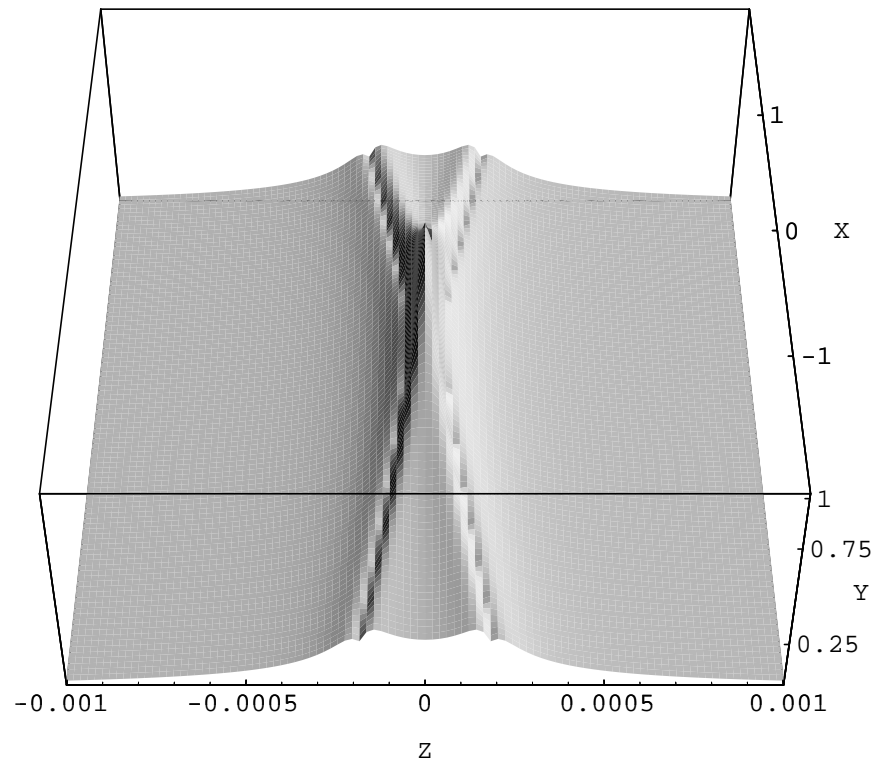


Figure 3: Exact X wave solution of homogeneous wave equation. The solution is cylindrically symmetric around the z axis; what we show here is the value of its real part on the plane $y = 0$.

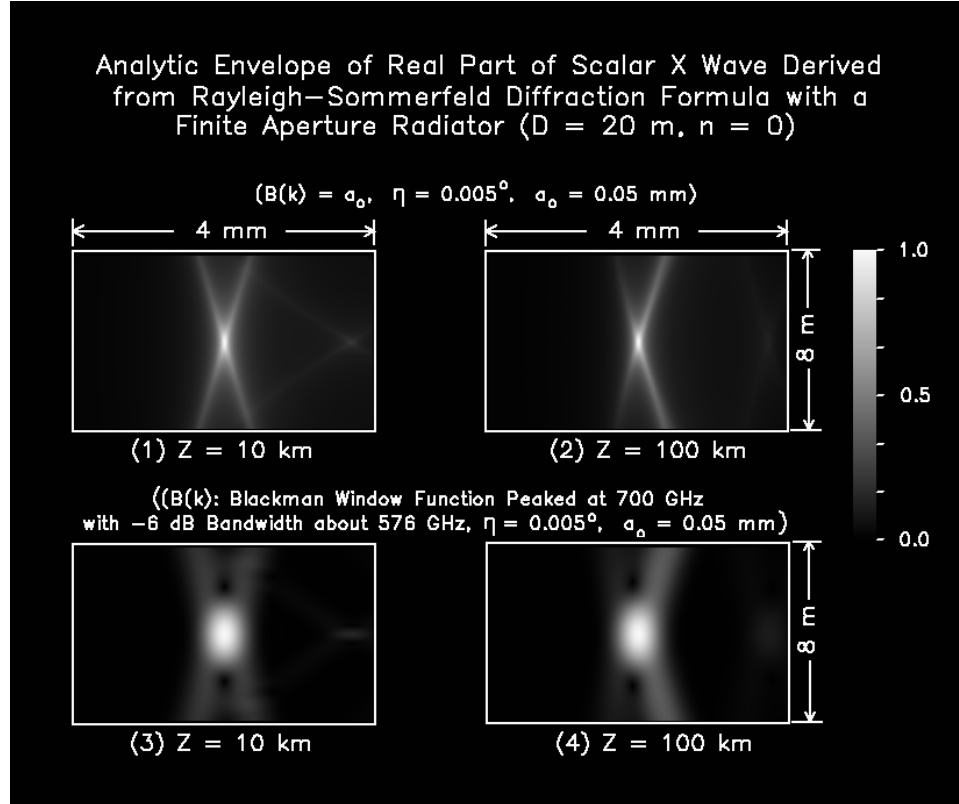


Figure 4: Aspect of an X wave propagating in the z direction. The pictures in (1) and (2) show an approximate solution obtained from the Rayleigh-Sommerfeld diffraction formula for a broad band X wave; in (3) and (4) we have the same kind of approximation for a band limited X wave. The latter might in principle be produced with present day technology. (Reprinted from [9].)

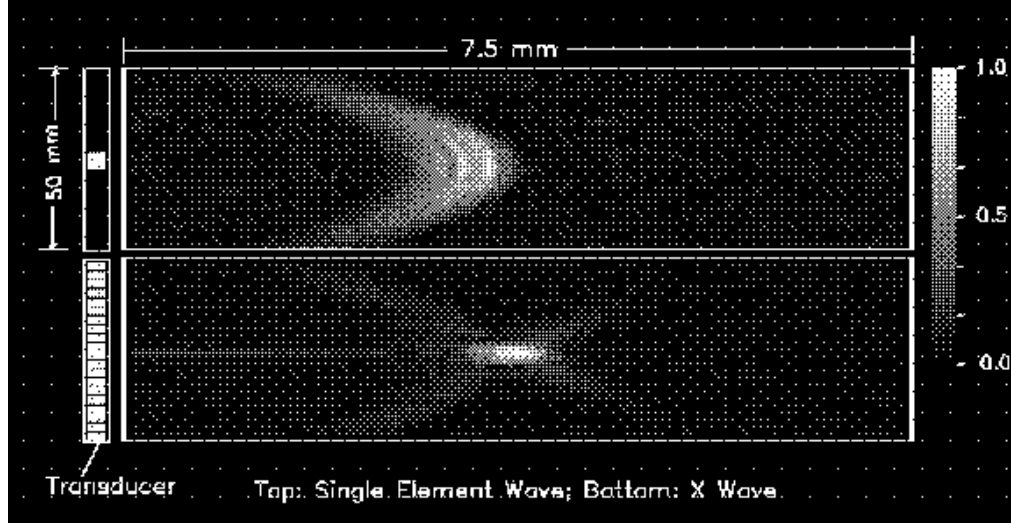


Figure 5: Comparison of the velocities of a single element acoustic wave and an acoustic X wave. (Reprinted from [9].)

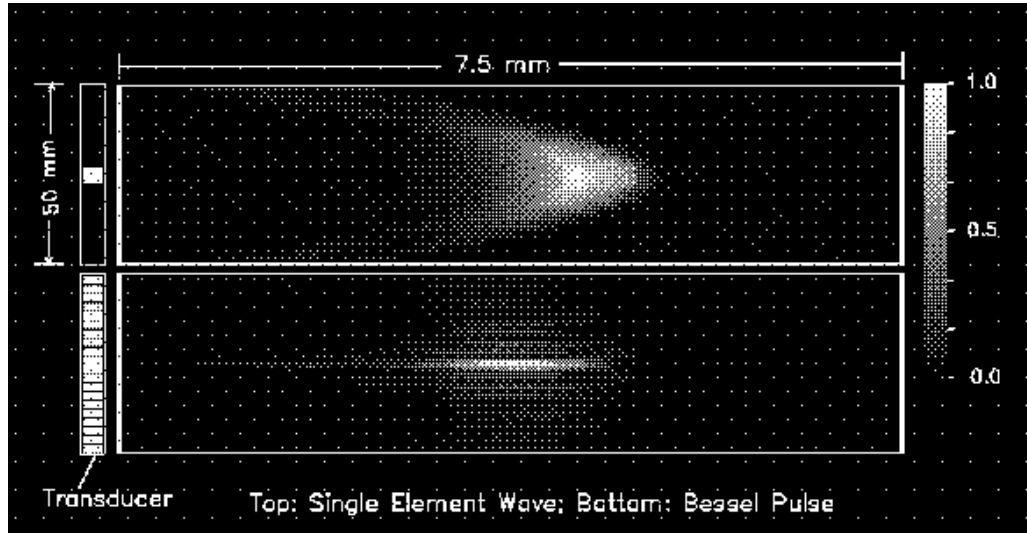


Figure 6: Comparison of the velocities of a single element acoustic wave and an acoustic Bessel pulse. (Reprinted from [9].)

W. Band [14]. In 1988, P. T. Papas and A. G. Obolensky claimed to have sent signals through coaxial cables with velocities up to 100 times greater than that of *LSs* [15]. Few people believe in these results, but a generalization of Band's theory predicts the possibility of propagation of superluminal modes in coaxial cables under appropriate boundary conditions.

- (iii) Besides these facts, there exist evidences that microwaves have been launched through horn antennas in the air with velocities around $1.47c$ for distances of about 1 m. For details see [16, 17, 18].

Another important theoretical consideration is the following. For more than half a century the problem of tunneling of wave packets through potential barriers has been investigated in countless articles. This problem is an important one since the tunneling of elementary particles of matter is a nontrivial prediction of quantum mechanics, responsible by the functioning of several semiconductor devices which are fundamental for modern technology.⁷ One conclusion of these works is that in the case of the tunneling of electromagnetic wave packets (which is formally equal to the quantum mechanical problem) the potential barrier can be physically realized by a special wave guide where there naturally occur certain modes of propagation called "evanescent".

Several recent experiments have confirmed the superluminal propagation through barriers. These results have been published in prestigious periodicals such as *Physical Review Letters*, *Physics Letters A*, *Journal of Applied Physics* and others. In particular, R. Chiao and his collaborators, from Berkeley University, could observe a single photon going through a barrier with a velocity 1.47 times the velocity of *LSs* [20]. G. Nimtz [21] transmitted Mozart's symphony #40 between two points 11.7 cm apart with a velocity 4.7 times that of *LSs*.

We must remark here that the always quoted results from Sommerfeld and Brillouin [22] showing that electromagnetic waves cannot travel through a dispersive medium with velocity greater than c is valid only for *ideal* (or mathematical) signals which have discontinuities in their first (or second) derivatives and are transverse waves. In such cases, as is well known ([23], vol. 2, p. 178), the signal must propagate along the light-cone characteristics of Maxwell equations.⁸ As correctly pointed out by Nimtz [21], ideal signals

⁷For a revision see [19].

⁸Maxwell equations have more general characteristics other than the light-cone. See [23].

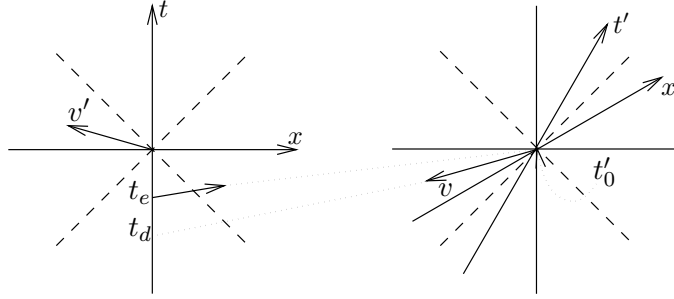


Figure 7: Illustration of Einstein's reasoning. The superluminal signal from S' would arrive at S before the signal from S was sent.

cannot be produced in practice, and for *real* signals the Brillouin-Sommerfeld results do not apply.

These spectacular achievements have given rise to much discussion. Are such results incompatible with the theory of relativity ? Have we found its limit of validity ?

Well, we may read in every textbook on special relativity, and also on research articles, that the theory of relativity implies that no *signal* may propagate with velocity *greater* than the velocity of *LSs* in vacuum. This claim is known as *causality principle* and its acceptance is due mainly to an argument of Einstein. We shall consider here a more opportune version of the argument.

Consider two inertial frames S and S' moving with velocity V with respect to each other (see fig. 7) and suppose that the observers in S and S' can produce electromagnetic X waves with velocity $v > c$, as measured in their respective reference frames, where all clocks have been synchronized 'à l'Einstein'. Suppose also that the observer in S has agreed with his friend in S' to make the following experiment: "If you receive a signal coming from my laboratory until hour zero of your clock (time t'_0 in fig. 7), you should destroy my laboratory with an X wave of velocity $v > c$, as we have agreed." It can be shown that if S' receives the signal from S (which was sent at time t_e in fig. 7) and uses his launcher of X waves, he will be able to destroy the laboratory in S at an instant t_d *earlier than* t_e . This constitutes a logical paradox, and based on such a reasoning Einstein concluded that *there is no propagation of superluminal signals*.

Einstein's conclusion does not resist a more careful analysis. Indeed, in order for this conclusion to be correct it is necessary that relativity theory it-

self be valid. But the existence of superluminal signals implies that there are processes for the synchronization of clocks inside an inertial reference frame which do not agree with the synchronization à l'Einstein. In particular, the existence of a transcendent superluminal signal (as is the case for some of the tunneling experiments) may eventually be used to effect a Newtonian synchronization, i.e. the type of synchronization supposed true by classical physics, what would imply a falsification of the principle of relativity [24].

There are many delicate and subtle points about these questions which cannot be discussed here. We would like to call the readers' attention to the fact that the claim that there is no contradiction between the principle of relativity and the existence of superluminal signals, as maintained by some authors (e.g. [25]), is false. For a more thorough discussion of this problem the reader may look at [9].

Before we finish we would like to remark that even 150 years after their discovery, Maxwell's equations are still a source of great surprises. Indeed, in a series of recent articles ([10, 26, 27, 28]) it has been verified that there exists an unexpected relation between Maxwell's equations and the Dirac equation, a relation which also unveils the geometric origin of the so called supersymmetry between bosons and fermions.

We believe that the experiments mentioned above, and a series of other experiments involving the foundations of quantum mechanics (discussed in the book [29]), are the first clouds appearing in the horizon of physics at this end of century. And we believe that they will surely have unpredictably deeper consequences than those thought of by Lord Kelvin at the end of last century, when he referred to the experiments on the black-body radiation and the Michelson-Morley experiments.

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